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K.M. Shapen[1], **A.T. Rakhymova**[1*], **A.A. Akhmedov**[2]
[1]L.N. Gumilyov Eurasian National University, Astana, Kazakhstan
[2]Dallas College, Dallas, USA
*e-mail: aigerim_rakhimova@mail.ru

HIERARCHICAL BAYESIAN MODELING OF LATENT RANKING PROCESSES

This paper proposes a universal Bayesian hierarchical model for estimating latent ranking processes in e-commerce search engines. Unlike traditional regression approaches, ranking is considered as an ordinal manifestation of an unobserved rating, the dynamics of which are described by a random walk with price drift. The model combines an equation of state, an ordinal logistic model of observations, and hierarchical pooling of sensitivity across product categories. The architecture is invariant to the platform and the structure of the algorithm's features. The proposed approach allows for the time dependence of observations to be taken into account and the latent state of the system to be reconstructed without access to internal ranking mechanisms, significantly expanding the capabilities of quantitative analysis of digital platform search algorithms. The estimation is performed entirely in a Bayesian setting using the MCMC (NUTS) method, obtaining posterior distributions of the parameters and hidden states of change. Empirical verification was conducted on a proprietary panel sample of search results generated by the monitoring system. The obtained results confirm the effectiveness of the model for reconstructing hidden rankings, identifying heterogeneity in price sensitivity between categories, and predicting position changes without access to the ranking algorithm.

Keywords: Bayesian modeling, latent variables, ranking, e-commerce, hierarchical models.

Қ.М. Шапен[1], А.Т. Рахымова[1*], А.А. Ахмедов[2]
[1]Л.Н. Гумилев атындағы Еуразия ұлттық университеті, Астана, Қазақстан
[2]Даллас колледжі, Даллас, АҚШ
*e-mail: aigerim_rakhimova@mail.ru

Ранжирлеудің латентті процестерін иерархиялық байестік модельдеу

Бұл мақалада электрондық коммерция іздеу жүйелеріндегі латентті рейтинг процестерін бағалауға арналған әмбебап байес иерархиялық моделі ұсынылады. Дәстүрлі регрессиялық тәсілдерден айырмашылығы, рейтинг байқалмаған рейтингтің реттік көрінісі ретінде қарастырылады, оның динамикасы баға ауытқуымен кездейсоқ серуендеу арқылы сипатталады. Модель күй теңдеуін, бақылаулардың реттік логистикалық моделін және өнім санаттары бойынша сезімталдықтың иерархиялық жиынтығын біріктіреді. Архитектура платформа мен алгоритмнің ерекшеліктерінің құрылымына қатысты өзгермейді. Ұсынылған тәсіл бақылаулардың уақытқа тәуелділігін ескеруге және жүйенің латентті күйін ішкі рейтинг механизмдеріне қол жеткізбестен қалпына құруға мүмкіндік береді, бұл сандық платформа іздеу алгоритмдерінің сандық талдау мүмкіндіктерін айтарлықтай кеңейтеді. Бағалау толығымен байес жағдайында MCMC (NUTS) әдісін қолдана отырып жүзеге асырылады, параметрлердің апостериорлық үлестірімдері мен латентті өзгерістерін алады. Эмпирикалық тексеру мониторинг жүйесімен жасалған іздеу нәтижелерінің меншікті панельдік үлгісінде жүргізілді. Алынған нәтижелер жасырын рейтингтерді қалпына құру, санаттар арасындағы баға сезімталдығының гетерогенділігін анықтау және рейтинг алгоритміне қол жеткізбестен позиция өзгерістерін болжау үшін модельдің тиімділігін растайды.

Түйін сөздер: байестік модельдеу, латентті айнымалылар, ранжирлеу, электрондық коммерция, иерархиялық модельдер.

К.М. Шапен[1], А.Т. Рахымова[1*], А.А. Ахмедов[2]
[1]Евразийский национальный университет имени Л.Н. Гумилева, Астана, Казахстан
[2]Даллас колледж, Даллас, США
[*]e-mail: aigerim_rakhimova@mail.ru

Иерархическое байесовское моделирование латентных процессов ранжирования

В данной работе предлагается универсальная байесовская иерархическая модель для оценки скрытых процессов ранжирования в поисковых системах электронной коммерции. В отличие от традиционных регрессионных подходов, ранжирование рассматривается как порядковое проявление ненаблюдаемого рейтинга, динамика которого описывается случайным блужданием с изменением цены. Модель объединяет уравнение состояния, порядковую логистическую модель наблюдений и иерархическое объединение чувствительности по категориям товаров. Архитектура инвариантна к платформе и структуре признаков алгоритма. Предложенный подход позволяет учитывать временную зависимость наблюдений и восстанавливать скрытое состояние системы без доступа к внутренним механизмам ранжирования, что значительно расширяет возможности количественного анализа алгоритмов поиска на цифровых платформах. Оценка выполняется полностью в байесовской постановке с использованием метода MCMC (NUTS), получая апостериорные распределения параметров и скрытые состояния изменения. Эмпирическая проверка была проведена на собственной панельной выборке результатов поиска, сгенерированных системой мониторинга. Полученные результаты подтверждают эффективность модели для восстановления скрытых ранжирований, выявления неоднородности ценовой чувствительности между категориями и прогнозирования изменений позиций без доступа к алгоритму ранжирования.
Ключевые слова: байесовское моделирование, латентные переменные, ранжирование, электронная коммерция, иерархические модели.

1 Introduction

The digital transformation of retail between 2020 and 2025 has led to the formation of a highly concentrated e-commerce market structure in most countries, including the Republic of Kazakhstan. The market is taking on the characteristics of an oligopoly, dominated by ecosystem platforms that control access to demand. One manifestation of this global trend is the Kazakhstan market, where, according to the Bureau of National Statistics of the ASPR RK, the volume of retail e-commerce in 2024 reached 3.16 trillion tenge, which amounted to 14.1% of the total retail trade volume, with 84.9% (2.68 trillion tenge) coming from marketplaces. In the automotive spare parts segment, the Kaspi.kz platform holds a dominant position with an estimated share of 71-74%, making it a representative testing ground for studying general ranking patterns. Under such a concentrated market, product position in search results becomes a key intangible asset for the seller, determining revenue to a greater extent than traditional price and quality factors [1–5].

The position effect is universal across all e-commerce platforms and is described by a power-law relationship between rank and click-through probability. According to industry measurements from SISTRIX (28.5% in first position, 15.7% in second, 2.5% in tenth) and First Page Sage (39.8%, 18.7%, and 1.6%, respectively), confirming the findings of the researchers in [5], results beyond the first page (position > 10) collectively receive less than 0.78-5% of traffic. Thus, a shift from 5th to 15th position is equivalent to a loss of up to 80% of potential clicks, with the price remaining unchanged. This effect is not due to the specific algorithm, but to fundamental user behavior patterns, making the task of ranking modeling strategically important [5–7].

Existing scientific research on ranking on digital platforms can be divided into three areas. The first area is described by Learning to Rank models from the field of information retrieval, where ranking is considered a supervised learning problem with known relevance features. The second direction is econometric research on the influence of price on position using the examples of Amazon and Wildberries, which comes down to the assessment of static linear regressions of the type $\text{Position}_{it} = \alpha + \beta \ln \text{Price}_{it} + \epsilon_{it}$. Such works do not take into account the dynamic nature of ranking and the presence of a latent state of the system. The third approach is game-theoretic modeling of platform competition, which is primarily conceptual in nature and has not been empirically calibrated using panel data. These approaches lack a formalization of ranking as a stochastic process with unobservable state, which necessitates the development of a universal framework [8–12].

The aim of this study is to develop a universal mathematical framework for reconstructing unobserved product ranking trajectories and assessing their sensitivity to price changes based on price-position panel data. Unlike existing studies, the focus of this paper is not on constructing a model with maximum predictive accuracy, but on probabilistically reconstructing the latent state of the ranking algorithm and quantifying the uncertainty of the resulting conclusions. This approach allows us to interpret the search results generation mechanism as a dynamic process with a hidden state, rather than as a static relationship between price and observed position.

To achieve this goal, the following tasks were completed:

- A search results monitoring system for an electronic platform was developed and implemented, enabling high-frequency data collection. The system was tested on a representative sample of auto parts queries with a frequency of two observations per day.

- The ranking process was formalized as a Bayesian hierarchical state-space model with a latent variable describing the hidden rating.

- A parameter estimation algorithm based on the Hamiltonian Monte Carlo method was proposed, using the No-U-Turn Sampler algorithm, which ensures efficient sampling in high-dimensional spaces.

- A computational experiment was conducted on proprietary datasets containing over 6,000 observations for June-July 2026.

- An analysis of price sensitivity heterogeneity between product categories was performed [11, 13, 14].

The scientific novelty of the study is a creation of universal formalization of ranking as a stochastic process with unobservable state, where position is considered not as a dependent variable, but as an ordinal manifestation of a latent rating. A hierarchical model has been developed that combines three elements that have not previously been applied together for marketplaces: state dynamics in the form of a Gaussian random walk, an ordinal logistic observation model, and partial parameter pooling. A method for identifying individual sensitivity β_i under unobservable rating conditions is proposed, which allows one to estimate the elasticity of a position without access to the platform algorithm. A unique panel dataset has been created that demonstrates its suitability for calibrating complex Bayesian models. Unlike most existing studies, this work focuses not only on predicting product rankings but also on reconstructing latent rating dynamics that are not directly observable [11, 12, 15].

The methodology is based on Bayesian statistics, time series theory and panel data econometrics, and the estimation is performed in the PyMC.

2 Problem statement

Let, as a result of monitoring at moments $t = 1, \ldots, T$ for a set of goods $i = 1, \ldots, N$ an observation panel be formed:

$$\mathcal{D} = \{P_{it}, \text{Price}_{it}, q(i)\} \tag{1}$$

where P_{it} is the position, Price_{it} is the price, $q(i)$ is the request category.

The platform algorithm is an unknown function $\mathcal{A}$ such that:

$$P_{it} = \text{rank}\left(\mathcal{A}(X_{it})\right) \tag{2}$$

where X_{it} is the vector of internal characteristics of the product.

The problem is that the function $\mathcal{A}$ and the features X_{it} are unobservable, which makes direct estimation of the price impact impossible.

The hypothesis of the study is the existence of a scalar latent process S_{it} such that $\mathcal{A}(X_{it}) \approx S_{it}$, and the dynamics of S_{it} is recoverable from panel $\mathcal{D}$ (data from marketplace Kaspi).

It is required to construct a probabilistic model $p(S_{it}|\mathcal{D})$, estimate individual sensitivities $\beta_i = \frac{\partial S_{it}}{\partial \ln \text{Price}_{it}}$ and obtain the predictive distribution $p(P_{i,t+1}|\text{Price}_{i,t+1}, \mathcal{D})$.

The problem belongs to the class of inverse problems for systems with unobservable states and is solved in a Bayesian formulation [11, 13].

An empirical test of the proposed model was conducted using a proprietary search results data panel generated through automated monitoring of Kaspi.kz platform. The results obtained allow us to evaluate the applicability of the proposed approach for analyzing search ranking dynamics and determine the impact of price changes on the latent competitive position of products in various categories.

3 Methodology

To solve problem (1)-(2), consider the following Mathematical model of latent ranking dynamics. Let us consider a set of goods $i = 1, \ldots, N$ observed at time points $t = 1, \ldots, T$.

For each product at time t, the following are observed:

- product price Price_{it};
- position in search results $P_{it} \in \{1, \ldots, K\}$, where K is the maximum monitoring depth.

It is assumed that the observed position is determined by the unobserved (latent) rating of the product S_{it}, which characterizes its relative competitiveness. Since the value of S_{it} is not directly observable, the task is to reconstruct its trajectory from observed changes in price and position.

Further we consider a state model. The hidden rating is assumed to change gradually and depend on the relative change in price:

$$S_{it} = S_{i,t-1} + \beta_i \Delta \ln \mathrm{Price}_{it} + \eta_{it} \tag{3}$$

where $\Delta \ln \mathrm{Price}_{it} = \ln \mathrm{Price}_{it} - \ln \mathrm{Price}_{i,t-1}$, and $\eta_{it} \sim \mathcal{N}(0, \sigma_\eta^2)$.

According to the equation of state (3), the latent rating changes under the influence of the relative change in price and a set of unobservable factors. The parameter β_i characterizes the individual sensitivity of the rating of product i to price changes. The random component η_{it} describes the combined influence of unobservable factors: the competitive environment, user activity, and the internal mechanisms of the algorithm.

The initial state is given by:

$$S_{it} \sim \mathcal{N}(\mu_0, \sigma_0^2) \tag{4}$$

The initial distribution (4) completes the specification of the state model.

Since the observed position is an ordinal value, the relationship between the latent rating and the observed data is given by an ordinal logistic model:

$$Pr(P_{it} \leq k \mid S_{it}) = \frac{1}{1 + \exp(-(c_k - S_{it}))}, \quad k = 1, \ldots, K-1 \tag{5}$$

where $c_1 < c_2 < \cdots < c_{K-1}$ are unknown threshold parameters.

From the observation model (5), it follows that the probability of a product being ranked in a given position is determined by the latent rating value relative to a system of threshold values, which allows for the ordinal nature of the dependent variable to be accurately accounted for without assuming linearity of the distances between positions.

To account for differences between product categories, price sensitivity coefficients are defined hierarchically:

$$\beta_i \mid q(i) = q \sim \mathcal{N}(\mu_\beta^q, \tau_\beta^{q^2}) \tag{6}$$

where μ_β^q is an average price impact in category q, and τ_β^q is an intra-group heterogeneity.

Relation (6) introduces a hierarchical structure for the model, allowing for simultaneous consideration of general patterns within product categories and the individual characteristics of individual products. Weak priors are used for the hyperparameters:

$$\mu_\beta^q \sim \mathcal{N}(m_0, s_0^2), \quad \tau_\beta^q \sim \mathrm{Half-Normal}(\lambda) \tag{7}$$

The prior distributions (7) complete the structure of the hierarchical part of the model.

The joint posterior distribution of the model is:

$$p(S, \beta, \theta \mid P, Price) \propto p(P \mid S, \theta)\, p(S \mid Price, \beta)\, p(\beta)\, p(\theta) \tag{8}$$

where S is a set of latent states, β are the individual price sensitivity coefficients, and θ are hyperparameters of the model.

Equation (8) combines the state model, observation model, and prior information into a single probabilistic formulation. The estimation is performed by the MCMC method using the NUTS algorithm (PyMC).

To justify the need for latent dynamics, a comparison based on the expected predictive density (LOO-CV) is used [16], [17]. The elpd_loo criterion is the modern standard for hierarchical Bayesian models and is more robust than the WAIC. The comparison is performed on the same observed variable—position rank—so the values are comparable. A model with a larger elpd_loo has better predictive power.

The model allows us to estimate:

- the latent dynamics S_{it} for each product;
- individual sensitivity β_i;
- the distribution of sensitivities within categories;
- the uncertainty of estimates.

Unlike the classical regression $P = f(Price)$, the proposed approach considers the position as an observable manifestation of a hidden ranking process, which allows us to take into account temporal variability and obtain probabilistic forecasts.

Thus, state equation (3) describes the evolution of the latent rating, observation model (5) relates the latent state to the observed position, relations (6)–(7) define the hierarchical structure of the parameters, and joint distribution (8) combines all components into a single Bayesian model.

4 Results

The validation of the model was performed using data from marketplace Kaspi.kz. The initial monitoring covered 50 products of model Toyota Camry 70 across seven categories: oil, air, and cabin filters, spark plugs, batteries, wheel bearings, and control arms. The final modeling panel consisted of 5 641 observations across 32 time periods.

Results of computation confirm the convergence of the chains: r_hat = 1.00–1.01, the effective sample size has increased by 3.8 times — for μ_β oil filters from 277 to 1077, for most parameters > 1800 for ESS_$bulk$, which allows us to move on to a meaningful analysis.

Table 1: Convergence and posterior sampling efficiency diagnostics for the mu_beta price sensitivity hyperparameters

Category	Mean	Sd	r_hat	ess_bulk	ess_tail
Oil filter	0.141	0.096	1.00	1077.0	1556.0
Spark Plugs	0.018	0.064	1.01	497.0	970.0
Battery	0.010	0.174	1.01	906.0	1350.0
Wheel bearing	0.007	0.505	1.00	4201.0	2085.0
Cabin Filter	0.006	0.516	1.00	5313.0	2660.0
Suspension arm	-0.000	0.504	1.00	4198.0	2565.0
Air Filter	-0.001	0.510	1.00	4394.0	2435.0

The results of computation are presented in Figure 1.

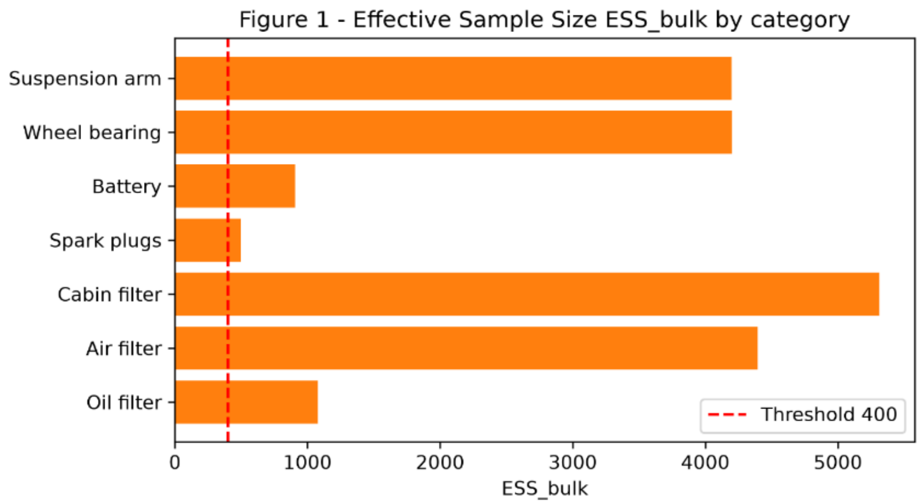

Figure 1: Effective sample size ESS_bulk by category.

All computation parameters exceed the threshold value ESS_bulk = 400, marked with a red dotted line, which indicates a sufficient effective size of the posterior sample and the reliability of the obtained estimates.

Thus, despite the presence of residual divergences, the final configuration ensures stable convergence of Markov chains, satisfactory values of diagnostic criteria and allows us to proceed to the analysis of the obtained estimates of the model parameters.

Let's move on to a comparison with the baseline models using the LOO criterion [16]. To assess the feasibility of complicating the model structure, two static specifications were initially considered, constructed on the same sample of 5,641 observations.

Values of the expected logarithmic prediction density (ELPD-LOO), the effective number of parameters (pLOO) and the difference of ELPD relative to the baseline specification.

Table 2: Comparison of the basic model and the hierarchical OrderedLogit model taking into account the product category according to the LOO-CV criterion [16]

Model	**elpd_loo**	**p_loo**	**elpd_diff**
Baseline log_price	-18 354.4	10.0	0.0
OrderedLogit category	-18 358.2	16.2	-3.8 $\pm$ 2.8

Note: elpd_loo is the expected logarithmic predictive density, p_loo is the effective number of parameters, elpd_diff is the difference relative to the baseline model. The difference of -3.8 ± 2.8 *is not statistically significant (*$|diff| < 2\,\mathrm{SE}$*), the category model does not improve predictive ability relative to the baseline log_price.*

The results show that introducing categorical coefficients does not significantly improve the model's predictive performance. The ELPD difference is only -3.8 for similar values of the effective number of parameters. Therefore, increasing the complexity of the static specification due to categorical heterogeneity does not, in itself, improve the model's performance. This suggests that the main reason for the limited explanatory power is not the absence of categorical effects, but the neglect of the temporal dynamics of the latent state.

Now we turn to assessing the price sensitivity of various product categories. The main result of the computational experiment is the a posteriori estimates of the hyperparameters μ_β, which characterize the average sensitivity of the latent product rating to price changes within each product category. Estimates revealed heterogeneity in the price effect shown in Figure 2. For six of the seven categories, the posterior means are near zero, and the 95% high-density intervals (HDIs) include zero, making it impossible to conclude that price has a statistically significant effect.

The exception is the oil filter category. This category yielded the most pronounced estimate of all the groups studied: $\mu_\beta = 0.141$ with a standard deviation of 0.096 and a 95% HDI of $[-0,036 \mid 0,328]$. Despite the partial zero-crossing, it is here that the most stable price signal is observed, likely due to intense competition.

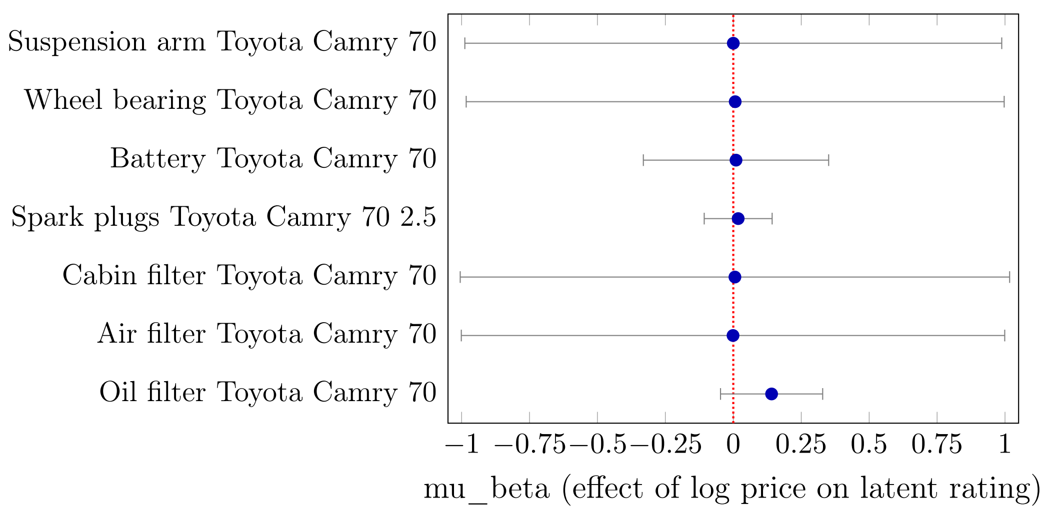

Figure 2: Post hoc estimates of mu beta hyperparameters for 7 product categories with $95\% HDI$.

Thus, the results confirm the heterogeneity of price influence across product categories. For most categories, no statistically significant relationship was found, whereas for oil filters, a consistent positive sensitivity of the latent rating to price changes was observed. Consequently, the ranking algorithm's response is determined not only by price per se, but also by the competitive structure of the relevant product category.

Let us now consider the individual coefficients and the effect of hierarchical pooling. The a posteriori estimates of the individual sensitivity coefficients β_i are concentrated in a relatively narrow range of 0.13–0.25. The highest estimate was obtained for the ATB filter set (oil SB2287, air SH4031, cabin SA1334, ID 147935379): $\beta = 0,255$ at 95% HDI $[0,001|0,504]$. The high-density intervals for all products overlap significantly, which indicates the limited information content of individual time series and justifies the use of hierarchical pooling.

An illustration of the hierarchical pooling effect for the most represented products is shown in Figure 3.

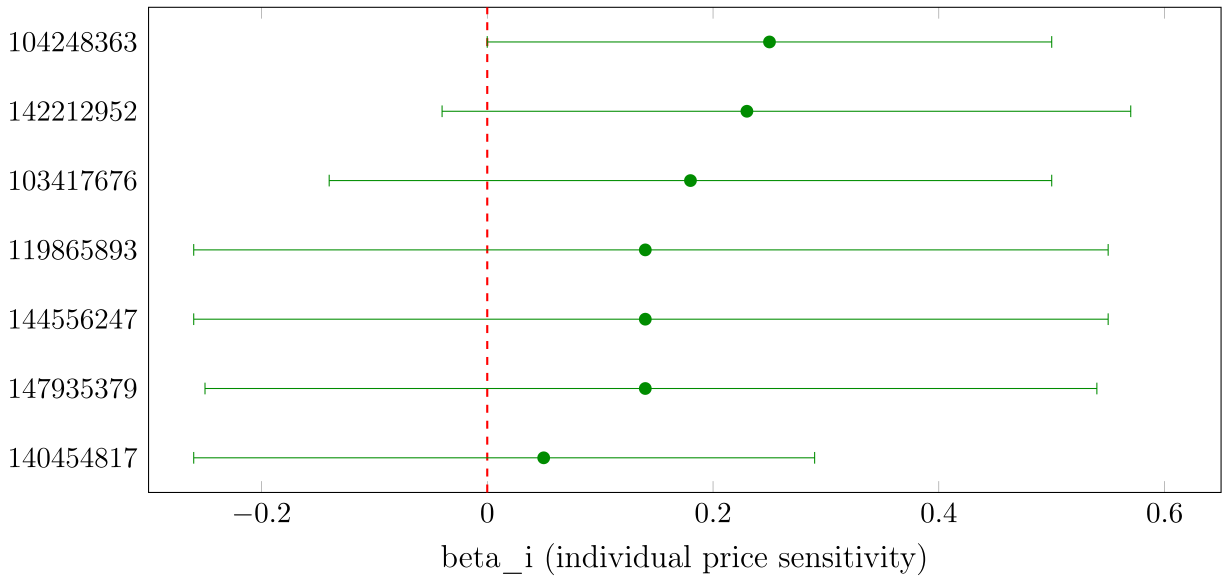

Figure 3: Individual sensitivity coefficients β_i for the TOP-7 products.

As the observation period increases, a gradual weakening of the pooling effect and a more pronounced differentiation of individual coefficients are expected. Therefore, the observed concentration of estimates around the group means reflects the correct performance of the model under conditions of limited data.

The key result of the model is the restoration of the latent dynamics of the rating S_t reflecting the internal state of the ranking process, in contrast to static approaches that operate only with the observed position

The reconstructed dynamics demonstrate a smooth change, indicating the presence of a stable hidden component. The product's position is shaped not only by the current price change but also by the previously accumulated state of the system. The state-space model allows us to separate these short-term and long-term effects, which is impossible in static models that assume independent observations (Figure 4).

As Figure 4 shows, the hidden states change consistently without sharp jumps, with stable confidence intervals. The 68% confidence interval is shown in the filled area. This demonstrates the correct reconstruction of the hidden component and allows us to separate systematic rating changes from random fluctuations, creating the basis for scenario analysis of the price's impact on future positions.

Based on the posterior distribution, scenarios for changes in the latent rating for price changes of -10%, -5%, -3%, and $+5\%$ were calculated. The impact is significantly heterogeneous across categories.

The most pronounced response is for oil filters; for other categories, the effect is less clear due to the wide ranges. This confirms the uneven sensitivity of product segments to price.

The practical value of the model lies in its quantitative assessment of changes in competitive positioning without field experiments or A/B testing on the platform. Although

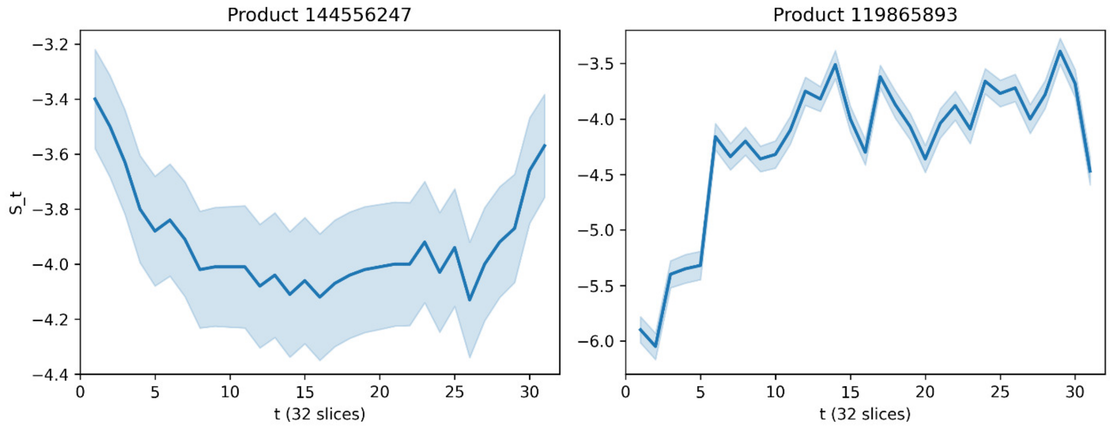

Figure 4: Examples of reconstructed dynamics of the latent rating S_t for two products (set of ATB filters) for 32 time slices (two observations per day).

the model was tested on Kaspi.kz, its structure is universal: the equation of state describes the general mechanism by which price influences hidden ratings, and the ordinal observation model is applicable to any system with discrete rank. The methodology is transferable to other marketplaces with panel data.

The results of the scenario analysis are presented in Fig. 5.

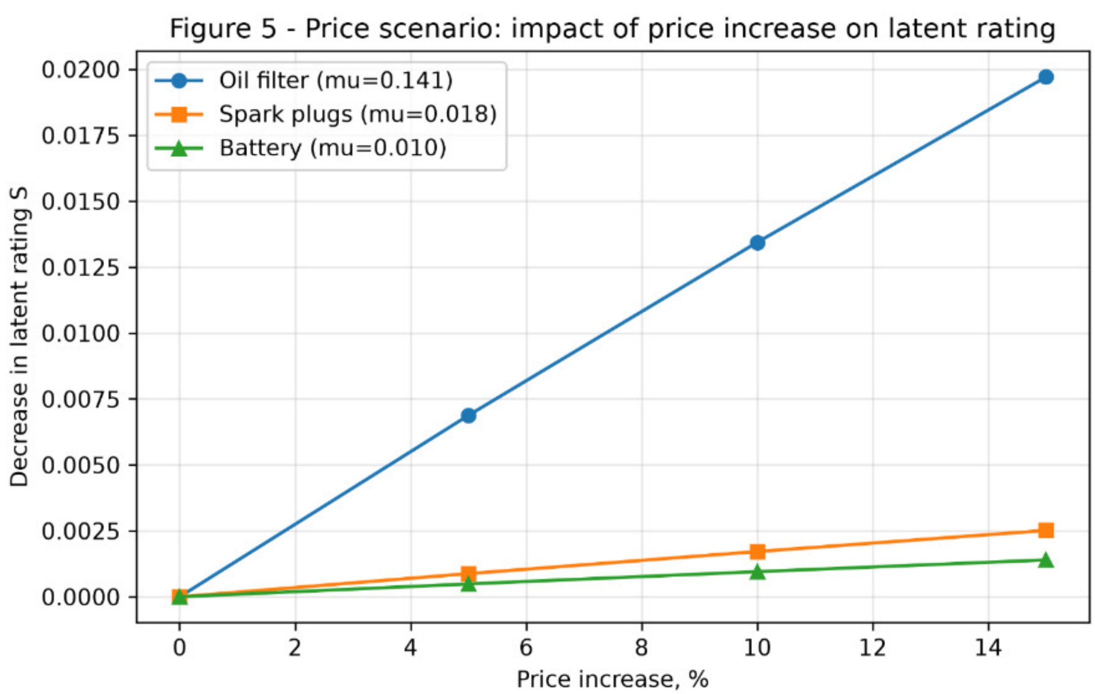

Figure 5: Forecasted decrease in the latent rating S_t under scenarios of price growth by 5%, 10%, 15% for three representative categories.

For oil filters, a 10% price increase reduces the rating by 0.013, which is equivalent to a drop in P(TOP5) from 65.0% to 64.7%. For spark plugs and batteries, the drop is only 0.0017 and 0.0009 ($\Delta P < 0.1$ pp).

Table 3: Calculation of the price effect in P(TOP5) for highly sensitive product "103417676" ($\beta_i = 0.255$), base $P_0 = 65\%$

Price	**dS**	**P(TOP5)**
−10%	−0.0269	65.6%
−5%	−0.0131	65.3%
0%	0	65.0%
+5%	+0.0124	64.7%
+10%	+0.0243	64.4%

Thus, the model serves not only to explain the ranking mechanism, but also as a tool for predicting the consequences of pricing policy.

5 Discussion

The obtained results confirm the viability of the proposed Bayesian state-space model for reconstructing the latent dynamics of competitive position. Unlike static ranking models and ordinal regressions, the proposed approach describes the evolution of the latent rating as a Markov process, allowing one to separate the influence of short-term price changes from the accumulated latent state of the product.

Comparison with static ordinal models showed that considering only categorical heterogeneity does not lead to improved prognostic performance (ΔELPD $= -3.8$). Therefore, increasing the complexity of the static model alone is insufficient to improve prediction quality, and the role of dynamic latent state requires further study.

The reconstructed latent trajectories are characterized by smooth dynamics compared to the observed position changes, which indicates the presence of an inertial latent state that determines the ranking process.

Hyperparameter estimates revealed heterogeneity in price sensitivity across categories. For most categories, the high-density 95% intervals include zero, while oil filters showed the most pronounced positive sensitivity coefficient estimate, which may be due to higher competition in this category.

The practical significance of the model lies in the possibility of conducting scenario analysis without interfering with the platform's operation, allowing one to assess the risk of changes in the competitive position under various pricing policy options. The results of Bayesian inference confirm the acceptable quality of the estimates: for the main parameters, r_hat does not exceed 1.01, and the effective sample size ensures the stability of the posterior estimates.

6 Conclusion

This paper proposes a Bayesian hierarchical search ranking model that combines a state-space model and an ordinal logistic observation model. The proposed approach considers the observed position as a manifestation of a hidden dynamic ranking and allows for reconstructing its evolution from panel data.

The results of the computational experiment showed that statistically significant price sensitivity was observed only for certain product categories, while for most categories the impact of price changes in the period under review was not detected.

This paper proposes the development of a unified probabilistic model that combines latent state dynamics, hierarchical modeling of categorical heterogeneity, and an ordinal observation model. The proposed framework allows for the simultaneous reconstruction of latent competitive position dynamics, assessment of categorical price sensitivity, and consideration of the ordinal nature of the observed data.

Authors contribution

K.Sh. and A.A.: contributed to methodology and supervision; K.Sh. and A.R.: prepared the original draft. All authors have read and agreed to the published version of the manuscript.

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Information about authors:

Shapen Kuanysh – L.N. Gumilyov Eurasian National University, PhD student (Astana, Kazakhstan, email: shapen_kuanysh@mail.ru).

Rakhymova Aigerim (corresponding author) – PhD, L.N. Gumilyov Eurasian National University, Senior lecturer of the Department of Mathematical and Computer Modeling (Astana, Kazakhstan, email: aigerim_rakhimova@mail.ru).

Akhmedov Anvarjon – PhD, Professor at Harmony Science Academy, Dallas College (Dallas, USA, email: aakhmedov@dcccd.edu).

Авторлар туралы ақпарат:

Шапен Қуаныш Мұратұлы – Л.Н. Гумилев атындағы Еуразия ұлттық университеті, PhD докторанты (Астана, Қазақстан, электрондық пошта: shapen_kuanysh@mail.ru).

Рахымова Айгерім Тұрлыбайқызы (корреспондент автор) – PhD, Л.Н. Гумилев атындағы Еуразия ұлттық университеті, Математикалық және компьютерлік модельдеу кафедрасының аға оқытушысы (Астана, Қазақстан, электрондық пошта: aigerim_rakhimova@mail.ru).

Ахмедов Анваржон – Даллас колледжінің Harmony Science Academy, PhD профессоры (Даллас, АҚШ, электрондық пошта: aakhmedov@dcccd.edu).

Информация об авторах:

Шапен Куаныш Муратулы – магистр, докторант Евразийского национального университета имени Л.Н. Гумилева (Астана, Казахстан, электронная почта: shapen_kuanysh@mail.ru).

Рахымова Айгерим Турлыбайкызы (корреспондент автор) – PhD, старший преподаватель кафедры Математического и компьютерного моделирования Евразийского национального университета имени Л.Н. Гумилева (Астана, Казахстан, электронная почта: aigerim_rakhimova@mail.ru).

Ахмедов Анварджон – PhD, профессор Академии наук гармонии Далласского колледжа (Даллас, США, электронная почта: aakhmedov@dcccd.edu).

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